Properties of Logarithms

NEW!

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Lesson Vocabulary
- Change of Base Formula

Objective To use the properties of logarithms

Dr. A S

T. U. S. K.

Things Dr. Anglin Says U Should Know

take note

Properties **Properties of Logarithms**

For any positive numbers m, n, and b where $b \neq 1$, the following properties apply.

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \log_b m$

Essential Understanding Logarithms and exponents have corresponding properties.

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Express each expression as a single logarithm.

(A) $\log_3 27 - \log_3 81$

$$\log_3 27 - \log_3 81 = \log_3\left(\frac{27}{81}\right) \quad \text{Quotient Property of Logarithms}$$

$$= \log_3\left(\frac{1}{3}\right) \quad \text{Simplify.}$$

$$= \log_3 3^{-1} \quad \text{Write using base 3.}$$

$$= -1\log_3 3 \quad \text{Power Property of Logarithms}$$

$$= -1 \quad \text{Simplify.}$$

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Express each expression as a single logarithm.

(B) $\log_5\left(\frac{1}{25}\right) + \log_5 625$

$\log_5\left(\frac{1}{25}\right) + \log_5 625 = \log_5\left(\frac{1}{25}\ \boxed{\cdot}\ 625\right)$ **Product** Property of Logarithms

$= \log_5 \boxed{25}$ Simplify.

$= \log_5 \boxed{5^2}$ Write using base 5.

$= \boxed{2} \log_5 5$ Power Property of Logarithms

$= \boxed{2}$ Simplify

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What is each logarithm expanded?

A $\log \frac{4x}{y}$

log 4x - log y — *Quotient Property*

log 4 + log x - log y — *Product Property*

B $\log_9 \frac{x^4}{729}$

$\log_9 x^4 - \log_9 729$ — *Quotient Property*

$4 \log_9 x + \log_9 729$ — *Power Property*

$4 \log_9 x + \log_9 3^9$ — *Prime factorization*

$4 \log_9 x + 3$

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Got It? **2.** What is each logarithm expanded?

a. $\log_3 \frac{250}{37}$

$\log_3 250 - \log_3 37$ *Quotient Property*

$\log_3 2(5)^3 + \log_3 37$ *Factorization*

$3 \log_3 2(5) + \log_3 x$ *Power Property*

$3 (\log_3 2 + \log_3 5) + \log_3 37$ *Product Property*

b. $\log_3 9x^5$

$5 \log_3 9x$ *Power Property*

$5 \log_3 9 + \log_3 x$ *Product Property*

$5 \log_3 3^2 + \log_3 x$

$5 (2) + \log_3 x = 10 + \log_3 x$

Properties of Logarithms

You have seen logarithms with many bases. The log key on a calculator finds $\log_{10}$ of a number. To evaluate a logarithm with any base, use the **Change of Base Formula**.

take note

Property Change of Base Formula

For any positive numbers m, b, and c, with $b \neq 1$ and $c \neq 1$,

$$\log_b m = \frac{\log_c m}{\log_c b}.$$

Here's Why It Works

$$\log_b m = \frac{(\log_b m)(\log_c b)}{\log_c b} \qquad \text{Multiply } \log_b m \text{ by } \frac{\log_c b}{\log_c b} = 1.$$

$$= \frac{\log_c b^{\log_b m}}{\log_c b} \qquad \text{Power Property of Logarithms}$$

$$= \frac{\log_c m}{\log_c b} \qquad b^{\log_b m} = m$$

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What is the value of each expression?

A $\log_{81} 27$

$$\frac{\log 27}{\log 81} = 0.75$$

B $\log_5 36$

$$\frac{\log 36}{\log 5} \approx 2.2265$$

Properties of Logarithms

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Got It? **3.** Use the Change of Base Formula. What is the value of each expression?

a. $\log_8 32$

$$\frac{\log 32}{\log 8} = \; ?$$

b. $\log_4 18$

$$\frac{\log 18}{\log 4} = \; ?$$

Properties of Logarithms

$$L(I) = 10\log\left(\frac{I}{I_0}\right)$$

Solve a Real-World Problem

During a concert, an orchestra plays a piece of music in which its volume increases from one measure to the next, tripling the sound's intensity. Find how many decibels the loudness of the sound increases between the two measures.

Let I be the intensity in the first measure. So $3I$ is the intensity in the second measure.

Increase in loudness $= L(3I) - L(I)$	Write the expression.
$= 10\log\left(\frac{3I}{I_0}\right) - 10\log\left(\frac{I}{I_0}\right)$	Substitute.
$= 10\left(\log\left(\frac{3I}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right)$	Distributive Property
$= 10\left(\log 3 + \log\left(\frac{I}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right)$	Product Property of Logarithms
$= 10\log 3$	Simplify.
≈ 4.77	Evaluate the logarithm.

So the loudness of sound increases by about 4.77 decibels.

Properties of Logarithms

ANY QUESTIONS

Properties of Logarithms

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Classwork:

Worksheet 16.1

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Write each expression as a single logarithm.

1. $\log_5 4 + \log_5 3$
   $\log_5 12$

2. $\log_6 25 - \log_6 5$
   $\log_6 5$

3. $\log_2 4 + \log_2 2 - \log_2 8$
   $\log_2 1$

☐ $2 \log x - 3 \log y$
$\log \frac{x^2}{y^3}$

☐ $\frac{1}{2} \log r + \frac{1}{3} \log s - \frac{1}{4} \log t$
$\log \frac{r^{\frac{1}{2}} s^{\frac{1}{3}}}{t^{\frac{1}{4}}}$

☐ $\log_3 4x + 2 \log_3 5y$
$\log_3 100xy^2$

☐ $(\log 3 - \log 4) - \log 2$
$\log \frac{3}{8}$

☐ $5 \log x + 3 \log x^2$
$\log x^{11}$

☐ $\log_6 3 - \log_6 6$
$\log_6 \frac{1}{2}$

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Expand each logarithm. Simplify if possible.

13 $\log xyz$

$\log x + \log y + \log z$

☐ $\log_2 \frac{x}{yz}$

$\log_2 x - \log_2 y - \log_2 z$

☐ $\log 6x^3y$

$\log 6 + 3 \log x + \log y$

16 $\log 7(3x - 2)^2$

$\log 7 + 2 \log (3x - 2)$

☐ $\log \sqrt{\frac{2rst}{5w}}$

$\frac{1}{2} \log s + \frac{1}{2} \log t - \frac{1}{2} \log 5 - \frac{1}{2} \log w$

$\frac{1}{2} \log 2 + \frac{1}{2} \log r +$ ☐ $\log \frac{5x}{4y}$

$\log 5 + \log x - \log 4 - \log y$